Maximizing profit

Consider a market participant firm that operates under perfect competition. This firm sells its output at a constant market price p greater than zero, and incurs a cost w greater than zero for each unit of the single input it utilizes. The firm's production function is denoted by $f(x)$, with x being the amount of the variable input used. We aim to determine the input level x that will maximize the firm's profit. The production function is given by $f(x) = x$.

1. Write the profit function and show that is concave.
2. Find the critical points (if there are any) assuming $p = w$ and $p \neq w$.

Solution

1. The profit Π of a competitive firm as a function of input x can be modeled by the expression

$$\Pi(x) = px - wx$$

Given a function Π which is sufficiently smooth, specifically being twice-differentiable when $x > 0$, we examine its properties. The first derivative of the function, Π', is calculated as $p - w$, and the second derivative, Π'', is found to be 0. Since the second derivative is 0 for all positive x, the function Π exhibits concavity and convexity over the domain $x > 0$.

2. When the market price p equals the input cost w, each value of x constitutes a stationary point. Profit function would be $\Pi = 0$ and $\Pi' = 0$. Consequently, every positive x becomes a global maximizer of profit. The profit remains constant for all x, making $x = 0$ also a global maximizer.

   If the market price p diverges from the input cost w, there are no stationary points.

$$\Pi' = p - w$$

   Won't be 0 for any value of x. There are no global maximizers for $x > 0$. But the profit at $x = 0$ is zero, therefore, when $p < w$, this point maximizes profit since the profit for $x > 0$ would be negative. Conversely, if $p > w$, the profit grows unbounded as x increases, indicating no single value of x maximizes the firm's profit.